

## Soft gamma-ray repeaters and anomalous X-ray pulsars as highly magnetized white dwarfs

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We show that the soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) can be explained as recently proposed highly magnetized white dwarfs (B-WDs). The radius and magnetic field of B-WDs are perfectly adequate to explain energies in SGRs/AXPs as the rotationally powered energy. While the highly magnetized neutron stars require an extra, observationally not well established yet, source of energy, the magnetized white dwarfs, yet following Chandrasekhar's theory (C-WDs), exhibit large ultra-violet luminosity which is observationally constrained from a strict upper limit.

*Keywords:* white dwarfs; stars: magnetars; stars: magnetic field; gravitation; relativity

### 1. Introduction

Soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are, as of now, most popularly hypothesized as isolated, spinning down, highly magnetized neutron stars (NSs) (magnetar model)<sup>1</sup>. A NS of radius 10km with surface and central magnetic fields respectively  $B_s \sim 10^{14}$ G and  $B_c \sim 10^{16}$ G can have magnetic energy  $\sim 10^{48}$ erg, which could produce the luminosity  $\sim 10^{36}$ erg/sec, as observed for AXPs/SGRs, in its typical age. However, there are certain shortcomings in it. First of all, as of now, there is no evidence for a strongly magnetized NS — as strong as required for the magnetar model. Second, recent Fermi observations are inconsistent with predicted high energy gamma-ray emissions in the magnetars. Third, inferred upper limit of  $B_s$  for some magnetars, e.g. SGR 0418+5729, is quite smaller than the field required to explain observed X-ray luminosity. Fourth, the attempt to relate magnetars to the energies of supernova remnants or the formation of black holes is not viable. There are many more. These observations imply that the high magnetic dipole moment is not a mandatory condition for a magnetar.

Recently AXPs/SGRs have been argued<sup>2</sup> to be magnetized white dwarfs (WDs), following the idea proposed decades back<sup>3,4</sup>. Due to their larger radius, the rotationally powered energy for WDs could be quite larger than that for NSs. Hence, these authors attempted to explain the energy released in AXPs/SGRs through the occurrence of glitch and subsequent loss of the rotational energy. While this WD based model (hereinafter C-WD) does not need to invoke extraordinarily strong, unconfirmed observationally yet, magnetic field, it is challenged by the observed short spin periods (e.g. Ref. 5). In addition, due to larger radius, they should exhibit larger ultra-violet (UV) luminosities, which however, suffer from a deep up-

per limits on the optical counterparts (e.g. Refs. 5, 6) of some AXPs/SGRs, e.g. SGR 0418+5729.

Recently, Mukhopadhyay and his collaborators, in a series of papers, have proposed for the existence of highly magnetized WDs (see, e.g., Refs. 7, 8, 9, 10, 11) with mass significantly super-Chandrasekhar.  $B_s$  of such WDs could be as high as  $10^{12}\text{G}$  and  $B_c$  could be 2 – 3 orders of magnitude higher. These WDs (hereinafter B-WDs) are significantly smaller in size compared to their ordinary counterparts (e.g. polar with  $B_s \sim 10^9\text{G}$ ). Their radius could even be an order or order and half of magnitude higher than that of a NS. As the surface temperatures of WDs with different magnetic fields are not expected to differ significantly<sup>12</sup>, smaller the radius, smaller the luminosity of the WD is. Therefore, B-WDs should be consistent with the UV-luminosity ( $L_{UV}$ ) cut-off in AXPs/SGRs. Moreover, their typical  $B_s$  is consistent with observations, but adequate to explain AXP/SGR energies as rotationally/spin-down powered energy, unlike the NS based models and C-WDs.

Here we explore AXPs/SGRs as B-WDs. Although the evolution of B-WDs was argued by accretion, they may appear as AXPs/SGRs at the exhaustion of mass supply after significant evolution. Such WDs'  $B_s$  and  $R$  combination can easily explain AXPs/SGRs as rotationally powered WDs. All the machineries implemented in the magnetar model can be applicable for B-WDs as well, however, with a smaller  $B_s$  which is physically more viable. Hence, under the B-WD model, one does not necessarily need to invoke an extra-ordinarily source of magnetic energy — everything comes out naturally. We also show that the magnetic fields in B-WDs are in accordance with the virial theorem, subject to its modification based on the magnetic pressure in the magnetostatic condition.

## 2. Modelling magnetized white dwarfs as rotating dipoles

The rate of energy loss from an oscillating magnetic dipole is

$$\dot{E}_{\text{rot}} = -\frac{\mu_0 \Omega^4 \sin^4 \alpha}{5\pi c^3} |m|^2, \quad (1)$$

when the variation of dipole moment  $m$  arises due to a magnetic dipole having inclination angle  $\alpha$  with respect to its rotational axis,  $\Omega$  is the angular frequency of the dipole,  $c$  the light speed. Assuming the rotating, magnetized compact stars to be rotating magnetic dipole, the dipole nature of magnetic field is expressed as

$$B = \frac{\mu_0 |m|}{2\pi R^3}, \quad (2)$$

when  $R$  is the radius of star. However, the above energy loss rate can be defined as the rate of rotational kinetic energy change of star with moment of inertia  $I$  as

$$I\dot{\Omega} = \dot{E}_{\text{rot}}, \quad (3)$$

which leads to

$$B_s = \sqrt{\frac{15c^3 I P \dot{P}}{\pi^2 R^6 \sin^2 \alpha}} G, \quad (4)$$

when  $P$  is the rotational period and  $\dot{P}$  the period derivative,  $I = Mk^2$ ,  $M$  the mass of the compact star. Note that  $k \propto R$  and the proportionality constant depends on the nature of the matter and its distribution in the star and shape of the star. This is the upper limit of  $B_s$ . As  $P$  and  $\dot{P}$  for AXPs/SGRs are known from observations,  $B_s$  can be computed for a given mass-radius ( $M-R$ ) relation when  $\alpha$  is a parameter. Once  $B_s$  is estimated for an observation, the rotational/dipole energy  $E_{\text{rot}}$  stored in the star can be computed. This further quantifies the maximum energy stored in it, if there is no other source as adopted in the magnetar model.

### 3. Explaining AXPs/SGRs

We consider nine AXPs/SGRs explained as B-WDs, listed in Table 1. We also assume the surface temperature of WDs to be  $T_{UV} \sim 10^4$  K with WDs to be semi-solid sphere/ellipsoid.

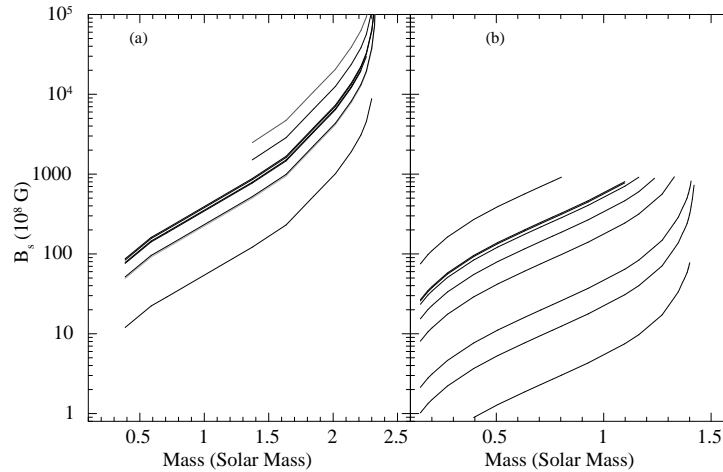


Fig. 1. Surface magnetic field as a function of mass for (a) B-WDs when from the top to bottom various curves correspond to SGR 1806-20, 1E 1048-59, SGR 0526-66, 1E 1841-045, 1E 1547-54, SGR 1900+14, 1E 2259+586, SGR 1822-1606, SGR 0418+5729, (b) C-WDs when from the top to bottom various curves correspond to SGR 1806-20, SGR 0526-66, 1E 1841-045, SGR 1900+14, 1E 1048-59, 1E 1547-54, 1E 2259+586, SGR 1822-1606, SGR 0418+5729.

Figures 1a and 1b show that  $B_s$  for the  $M-R$  combination of the B-WDs is quite stronger compared to that of the C-WDs. Also B-WDs exhibit X-ray luminosity:  $100 \lesssim \dot{E}_{\text{rot}}/L_x \lesssim 10^7$ , from Eq. (1), explaining all the AXPs/SGRs listed in Table 1 as rotational powered pulsars.

This naturally explains AXPs/SGRs without requiring an extra-ordinary source of magnetic energy. Generally, higher the  $B_c$  and/or  $B_s$  for a poloidal dominated field, higher the  $M$  is, which corresponds to a lower  $R$  and hence a lower  $L_{UV}$ .

Table 1. Various observational and theoretical parameters of AXPs/SGRs:  $P$ ,  $\dot{P}$ ,  $L_x$  are observed values and inputs and  $\alpha$ , minimum of  $L_{UV}$  are outputs of our model. See, <http://www.physics.mcgill.ca/~pulsar/magnetar/main.html>

AXPs/SGRs	$P$ (sec)	$\dot{P}$ ( $10^{-11}$ )	$L_x$ ( $10^{35}$ erg/sec)	$\alpha$ (degree)	$L_{UV \min}$ (erg/sec) B-WD	$L_{UV \min}$ erg/sec C-WD
1E 1547-54	2.07	2.32	0.031	5 – 15	$5.7 \times 10^{28}$	$4.8 \times 10^{29}$
1E 1048-59	6.45	2.7	0.054	5 – 15	$3.5 \times 10^{26}$	$9.2 \times 10^{29}$
1E 1841-045	11.78	4.15	2.2	15	$1.6 \times 10^{28}$	$1.7 \times 10^{30}$
1E 2259+586	6.98	0.048	0.19	2 – 3	$3.4 \times 10^{26}$	$1.5 \times 10^{29}$
SGR 1806-20	7.56	54.9	1.5	15	$3.4 \times 10^{26}$	$3.5 \times 10^{30}$
SGR 1900+14	5.17	7.78	1.8	15	$8.6 \times 10^{28}$	$1.3 \times 10^{30}$
SGR 0526-66	8.05	6.5	2.1	15	$6.4 \times 10^{27}$	$1.7 \times 10^{30}$
SGR 0418+5729	9.08	$5 \times 10^{-4}$	$6.2 \times 10^{-4}$	1 – 5	$3 \times 10^{28}$	$1.8 \times 10^{29}$
SGR 1822-1606	8.44	$9.1 \times 10^{-3}$	$4 \times 10^{32}$	1 – 5	$3.4 \times 10^{26}$	$8 \times 10^{28}$

#### 4. Maximum allowed magnetic field and modified virial theorem

First note very importantly that in the presence of strong magnetic field, the upper limit of magnetic fields in WDs, as discussed in, e.g. Ref. 13 for weak field cases, has to be revised, the contribution of the magnetic pressure to the hydro/magnetostatic balance equation cannot be neglected. Here we attempt to revise so in a simpler/approximate framework.

If the gravitational, thermal and magnetic energies are respectively denoted by  $W$ ,  $\Pi$  and  $\mu$ , then the scalar virial theorem can be read as

$$W + 3\Pi + \mu = 0, \text{ i.e. } -\alpha \frac{GM^2}{R} + 3M \frac{P}{\rho} + \frac{B^2}{24\pi} \frac{4}{3} \pi R^3, \quad (5)$$

where we consider, on average, the isotropic effects of averaged magnetic field  $B$  and hence overall the star to be spherical in shape,  $P$  is the pressure of the stellar matter,  $\rho$  the averaged density,  $M$  the mass of WD,  $G$  the Newton's gravitation constant. Now we assume that a polytropic equation of state (EoS) to be satisfied in entire star such that  $P = K\rho^\Gamma$ , where  $K$  and  $\gamma$  are the polytropic constants, and  $M = \frac{4}{3}\pi R^3\rho$ . Therefore, the scalar virial theorem can be reduced to

$$-\alpha \frac{GM^2}{R} + \beta \frac{M^\Gamma}{R^{3(\Gamma-1)}} + \gamma \frac{\Phi_M^2}{R}, \quad (6)$$

where  $\Phi_M = B\pi R^2$  and  $\alpha$ ,  $\beta$  and  $\gamma$  are the constant factors determined by the shape and other properties of the star.

Now combining first and second terms of Eq. (6), we obtain

$$M = \sqrt{\frac{\gamma\phi_M^2}{\alpha G \left(1 - \frac{\beta M^{\Gamma-2}}{\alpha G R^{3\Gamma-4}}\right)}}, \quad (7)$$

which is valid for any value  $\Gamma$ . For  $\Gamma = 4/3$ , it gives  $M = \sqrt{\frac{\gamma\phi_M^2}{\alpha G \left(1 - \frac{\beta M^{-2/3}}{\alpha G}\right)}}$  which is

independent of  $R$ , as expected from Chandrasekhar's theory, and needs to be solved for  $M$ . However, for  $\Gamma = 2$ , this gives  $M = \sqrt{\frac{\gamma\phi_M^2}{\alpha G(1-\frac{\beta}{\alpha GR^2})}}$ . Conversely, one can write  $R = \sqrt{\beta/\alpha G'}$ , where  $G' = G(1 - \gamma\phi_M^2/\alpha GM^2)$ .

Now in order to obtain the value of  $M$  and  $R$  explicitly, we have to evaluate the values of  $\alpha$ ,  $\beta$  and  $\gamma$ . The magnetostatic balance condition is given by

$$\frac{1}{\rho} \frac{dP}{dr} + \frac{1}{\rho} \frac{dP_B}{dr} = -\frac{Gm(r)}{r^2} \quad (8)$$

at an arbitrary radius  $r$  from the center of the star with mass enclosed at that radius  $m(r)$ , where  $\rho$  includes the contribution from  $B$  as well. We further assume the variation of  $B$  to be a power law like such that the corresponding magnetic pressure  $P_B = K_1 \rho_1^\Gamma$  with  $K_1$  and  $\Gamma_1$  being constants. Hence, the gravitation energy for this star is

$$\begin{aligned} W &= - \int_0^R \frac{Gm(r)}{r} 4\pi r^2 dr \rho = \int_0^R 4\pi r^2 dr \rho \frac{r}{\rho} \left( \frac{dP}{dr} + \frac{dP_B}{dr} \right) \\ &= -\frac{3(\Gamma_1 - 1)}{5\Gamma_1 - 6} \frac{GM^2}{R} + \frac{\Gamma - \Gamma_1}{5\Gamma_1 - 6} \frac{3\Pi}{\Gamma - 1}, \end{aligned} \quad (9)$$

assuming that  $\rho$  is negligibly small at  $r = R$ , surface of the star, compared to that at the center (or its averaged value). Therefore, from Eqs. (5) and (6), we obtain

$$-\frac{3(\Gamma_1 - 1)}{5\Gamma_1 - 6} \frac{GM^2}{R} + \left( 1 + \frac{\Gamma - \Gamma_1}{(5\Gamma_1 - 6)(\Gamma - 1)} \right) 3\Pi + \mu = 0 \quad (10)$$

and consequently

$$\alpha = \frac{3(\Gamma_1 - 1)}{5\Gamma_1 - 6}, \quad \beta = \left( 1 + \frac{\Gamma - \Gamma_1}{(5\Gamma_1 - 6)(\Gamma - 1)} \right) \frac{K 3^\Gamma}{(4\pi)^{\Gamma-1}}, \quad \gamma = \frac{1}{18\pi^2}. \quad (11)$$

An important outcome here is that  $\alpha$  is related to the scaling of  $B$  with  $\rho$ , which is indeed expected from the magnetostatic balance Eq. (8). In other words, the presence of magnetic pressure allows either a massive or/and smaller star. Obviously, for  $\Gamma = \Gamma_1$  the result reduces to that of the nonmagnetic case with the redefinition  $K$ .

Following Refs. 13, 2, the upper bound of  $B$  for a gravitationally bound star corresponds to

$$\mu = \frac{B^2 R^3}{18} = \frac{3(\Gamma_1 - 1)}{5\Gamma_1 - 6} \frac{GM^2}{R} \quad (12)$$

which leads to maximum allowed  $B$

$$B_{max} = \sqrt{\frac{54(\Gamma_1 - 1)}{(5\Gamma_1 - 6)} \frac{GM^2}{R^4}} = 7.829 \times 10^8 \frac{M}{M_\odot} \left( \frac{R_\odot}{R} \right)^2 \sqrt{\frac{\Gamma_1 - 1}{5\Gamma_1 - 6}} \text{G}, \quad (13)$$

where  $R_\odot$  is the radius of Sun. For a B-WD having  $B_c = 8.8 \times 10^{15} \text{G}$ , and hence averaged  $B_{max} = 4.4 \times 10^{15} \text{G}$ ,  $M = 2.44$  solar mass and  $R = 654 \text{km}$ , with  $\Gamma = 2$ , as reported in Refs. 7, 8, in order to satisfy Eq. (7),  $\Gamma_1$  has to be 1.2029. Similarly,

for the case of  $M = 1.77$  solar mass, equatorial  $R = 956.14\text{km}$ ,  $B_c = 5.34 \times 10^{14}\text{G}$ , with  $\Gamma \approx 4/3$  (Ref. 11),  $\Gamma_1$  has to be 3.5589. Importantly,  $P(\rho)$  profile and, hence,  $\Gamma$  is determined by  $P_B(\rho)$  profile, which however has not been strictly followed in this approximate calculation.

## 5. Conclusions

The present work indicates a possibility of wide application of recently proposed B-WDs in modern astrophysics. The idea that AXPs/SGRs need sources of energy other than rotational or accretion is certainly inevitable, but the hypothesis that they are highly magnetic NSs, although attractive, did not neatly fit in with further observations (unlike other ideas in Astrophysics, like, spinning NSs as radio pulsars and accreting compact objects as X-ray binaries, which quickly established themselves as paradigms). Hence, it is very important that other possible explanations for the AXP/SGR phenomena need to be seriously explored. The B-WD concept is an extremely attractive alternate for AXPs/SGRs.

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